Exam. Code : 103202
Subject Code : 1025

B.A./B.Sc. 2nd Semester
MATHEMATICS
Paper—I
(Calculus and Differential Equations)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Paper consists of *four* Sections A, B, C and D. Each section contains *two* questions. Students are required to attempt *five* questions, selecting at least *one* question from each section. The **fifth** question may be attempted from any section.

SECTION—A

1. (a) Find the intervals in which the curve $y = (\cos x + \sin x)e^x$ is concave upwards or concave downwards in $(0, 2\pi)$. Also find the points of inflexion.

   (b) Find the centre of curvature at any point (x, y) of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. Also find the evolute of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 5+5=10

2. (a) Find all the asymptotes of the curve $(x - y + 1)(x - y - 2)(x + y) = 8x - 1$.

   (b) Find the position and nature of the double points on the curve $y^2 = (x - 1)(x - 2)^2$. 5+5=10

SECTION—B

3. (a) Integrate $\int \sinh x \sinh 2x \sinh 3x\, dx$.

(b) Find the area of the region bounded by the curves $y^2 = 4a(x + a)$, $y^2 = 4b(b - x)$ where $a > 0$, $b > 0$. 5+5=10

4. (a) If $I_{m,n} = \int \sin^m x \cos^n x\, dx$ then prove that

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n+1} x}{m+1} + \frac{m+n+2}{m+1} I_{m+2,\, n}.$$ Hence

evaluate $\int \frac{dx}{\sin^4 x \cos^2 x}$.

(b) Find the length of a loop of the curve

$$9ay^2 = x(x - 3a)^2,\ a > 0$$ 5+5=10

SECTION—C

5. (a) Find the necessary and sufficient condition that the equation $Mdx + Ndy = 0$ may be exact where M, N are functions of x and y with the condition that $M, N, \frac{\partial M}{\partial y}, \frac{\partial N}{\partial x}$ are continuous function of x and y.

(b) Find the orthogonal trajectories of the system of circles touching a given straight line at a given point. 5+5=10

6. (a) Solve the differential equation

$$(8p^3 - 27)x - 12p^2y = 0 \text{ where } p = \frac{dy}{dx}$$

and investigate whether a singular solution exists.

(b) Solve $\frac{2y}{x} - p = f\left(\frac{p}{x} - \frac{y}{x^2}\right)$ where $p = \frac{dy}{dx}$.

5+5=10

SECTION—D

7. (a) Solve the differential equation

$$(D^4 + 2D^2 + 1)y = x^2 \cos x \text{ where } D = \frac{d}{dx}.$$

(b) Solve the differential equation

$(D^2 + 3D + 2)y = \sin(e^x)$ by the method of variation of parameters. 5+5=10

8. (a) Solve in series the differential equation

$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)\,y = 0$ where 2n is a non integer.

(b) Solve :

$$\sqrt{x}\,\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + 3y = x,\ x > 0.$$ 5+5=10